APPENDIX C

Calculations of Creep and Shrinkage Effects

See Design Step 5.3 for the basic information about creep and shrinkage effects. Design Step 5.3 also contains the table of fixed end moments used in this appendix.

Design Step C1.1

Analysis of creep effects on the example bridge

Calculations are shown for Span 1 for a deck slab cast 450 days after the beams are made. Span 2 calculations are similar. See the tables at the end of this appendix for the final results for a case of the slab and continuity connection cast 30 days after the beams are cast. All calculations are made following the procedures outlined in the publication entitled "Design of Continuous Highway Bridges with Precast, Prestressed Concrete Girders" published by the Portland Cement Association (PCA) in August 1969.

The distance from the composite neutral axis to the bottom of the beam is 51.54 in. from Section 2. Therefore, the prestressing force eccentricity at midspan is:

$$e_c = NA_{bottom} - CGS$$
$$= 51.54 - 5.0$$
$$= 46.54 \text{ in.}$$

Design Step C1.2

Calculate the creep coefficient, $\psi_{(t,\ ti)}$, for the beam at infinite time according to S5.4.2.3.2.

Calculate the concrete strength factor, k_f

$$k_f = 1/[0.67 + (f'_c/9)] \qquad\qquad (S5.4.2.3.2-2)$$
$$= 1/[0.67 + (6.0/9)]$$
$$= 0.748$$

Calculate the volume to surface area factor, k_c

$$k_c = \left[\frac{\left(\dfrac{t}{26e^{0.36(V/S)_b} + t}\right)}{\left(\dfrac{t}{45+t}\right)}\right]\left[\frac{1.80 + 1.77e^{-0.54(V/S)_b}}{2.587}\right] \qquad (SC5.4.2.3.2-1)$$

where:
    t = maturity of concrete
       = infinite days

    e = natural log base (approx. 2.71828)

$(V/S)_b$ = volume to surface ratio for the beam
= beam surface area is 2,955.38 in^2/ft (see Figure 2-3 for beam dimensions) and the volume is 13,020 in^3/ft
= (13,020/2,955.38)
= 4.406 in.

$$k_c = [1]\left[\frac{1.80 + 1.77e^{-0.54(4.406)}}{2.587}\right]$$

$$k_c = 0.759$$

The creep coefficient is the ratio between creep strain and the strain due to permanent stress (SC5.4.2.3.2)

Calculate creep coefficient according to Eq. S5.4.2.3.2-1.

$$\psi_{(\infty,1)} = 3.5k_ck_f(1.58 - H/120)t_i^{-0.118}[(t - t_i)^{0.6}/(10.0 + (t - t_i)^{0.6}]$$

where:
k_c = 0.759 (see above)
k_f = 0.748 (see above)
H = relative humidity
= 70%

t_i = age of concrete when load is initially applied
= 1 day

t = infinite days

$$\psi_{(\infty,1)} = 3.5(0.759)(0.748)(1.58 - 70/120)(1)^{(-0.118)}[1]$$
$$= 1.98$$

Design Step C1.3 Calculate the creep coefficient, $\psi_{(t,ti)}$, in the beam at the time the slab is cast according to S5.4.2.3.2.

t = 450 days (maximum time)

Calculate the volume to surface area factor, k_c

$$k_c = \left[\frac{\left(\dfrac{t}{26e^{0.36(V/S)_b} + t}\right)}{\left(\dfrac{t}{45 + t}\right)}\right]\left[\frac{1.80 + 1.77e^{-0.54(V/S)_b}}{2.587}\right] \qquad (SC5.4.2.3.2-1)$$

where:
  t   = 450 days
  e   = natural log base (approx. 2.71828)
  $(V/S)_b$ = 4.406 in.

$$k_c = \left[\frac{\left(\dfrac{450}{26e^{0.36(4.406)} + 450}\right)}{\left(\dfrac{450}{45 + 450}\right)}\right]\left[\frac{1.80 + 1.77e^{-0.54(4.406)}}{2.587}\right]$$

  k_c = 0.651

Calculate the creep coefficient, $\psi_{(t,ti)}$, according to Eq. S5.4.2.3.2-1.

  $\psi_{(450,1)} = 3.5k_ck_f(1.58 - H/120)t_i^{-0.118}[(t - t_i)^{0.6}/[10.0 + (t - t_i)^{0.6}]]$

  where:
    k_c = 0.651 (see above)
    k_f = 0.748 (see above)
    H   = 70%
    t_i = 1 day
    t   = 450 days

  $\psi_{(450,1)}$ = $3.5(0.651)(0.748)(1.58 - 70/120)(1)^{(-0.118)}[(450 - 1)^{0.6}/[10 + (450 - 1)^{0.6}]]$
      = 1.35

Calculate the restrained creep coefficient in the beam, ϕ, as the creep coefficient for creep that takes place after the continuity connection has been established.

  ϕ = $\psi_\infty - \psi_{450}$ (from PCA publication referenced in Step 5.3.2.2)
      = 1.98 – 1.35
      = 0.63

Design Step   **Calculate the prestressed end slope, θ.**
C1.4

For straight strands (debonding neglected). Calculate the end slope, θ, for a simple beam under constant moment.

  Moment = Pe_c

  θ = $Pe_cL_{span}/2E_cI_c$

where:

P = initial prestressing force after all losses (kips)

= 1,096 kips (see Design Step 5.4 for detailed calculations of the prestressing force)

e_c = 46.54 in. (calculated above)

L_{span} = 110.5 ft. (1,326 in.) (taken equal to the continuous beam span length)

E_c = the modulus of elasticity of the beam at final condition (ksi)

= 4,696 ksi

I_c = moment of inertia of composite beam (in^4)

= 1,384,254 in^4

θ = [1,096(46.54)(1,326)]/[2(4,696)(1,384,254)]

= 0.0052 rads

Design Step C1.5

Calculate the prestressed creep fixed end action for Span 1

The equation is taken from Table 5.3-9 for prestressed creep FEA, left end span, right moment.

FEM_{cr} = $3E_cI_c\theta/L_{span}$

= [3(4,696)(1,384,254)(0.0052)]/1,326

= 76,476/12

= 6,373 k-ft

End forces due to prestress creep in Span 1:

Left reaction   = $R1_{PScr}$

= FEM_{cr}/L_{span}

= (6,373)/110.5

= 57.7 k

Right reaction = $R2_{PScr}$

= $-R1_{PScr}$

= -57.7 k

Left moment   = $M1_{PScr}$

= 0.0 k-ft

Right moment = $M2_{PScr}$

= FEM_{cr}

= 6,373 k-ft

6,373 k-ft

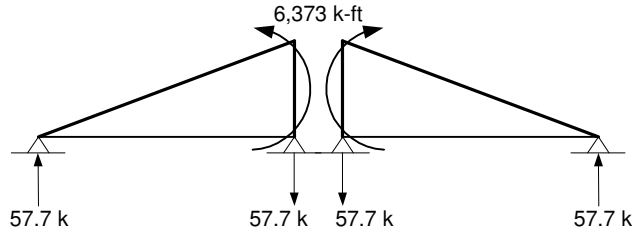57.7 k   57.7 k   57.7 k   57.7 k

Figure C1 – Presstress Creep Restraint Moment

Design Step C1.6

Calculate dead load creep fixed end actions

Calculate the total dead load moment at the midspan

Noncomposite DL moment = M_{DNC}
= 42,144 k-in (3,512 k-ft) (see Section 5.3)

Composite DL moment   = M_{DC}
= 4,644 k-in (387 k-ft) (see Section 5.3)

Total DL moment   = M_{DL}
= $M_{DNC} + M_{DC}$
= 42,144 + 4,644
= 46,788/12
= 3,899 k-ft

End forces due to dead load creep in Span 1:

Left reaction   = $R1_{DLcr}$
= M_{DL}/L_{span}
= 3,899/110.5
= 35.3 k

Right reaction = $R2_{DLcr}$
= $-R1_{DLcr}$
= -35.3 k

Left moment   = $M1_{DLcr}$
= 0.0 k-ft

Right moment = $M2_{DLcr}$
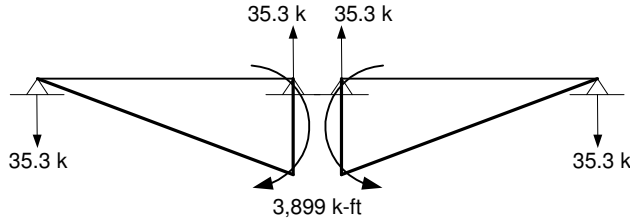= $-M_{DL}$
= -3,899 k-ft

Figure C2 – Dead Load Creep Restraint Moment

Calculate the creep correction factor, C_{cr}

$$
\begin{aligned}
C_{cr} &= 1 - e^{-\phi} \text{ (from PCA publication referenced in Step 5.3.2.2)} \\
&= 1 - e^{-0.63} \\
&= 0.467
\end{aligned}
$$

Calculate the total creep (prestress + dead load) fixed end actions for 450 days.

$$
\begin{aligned}
\text{Left reaction} \quad &= R1_{cr} \\
&= C_{cr}(R1_{PScr} + R2_{DLcr}) \\
&= 0.467(57.7 - 35.4) \\
&= 10.41 \text{ k}
\end{aligned}
$$

$$
\begin{aligned}
\text{Right reaction} &= R2_{cr} \\
&= -R1_{cr} \\
&= -10.41 \text{ k}
\end{aligned}
$$

$$
\begin{aligned}
\text{Left moment} \quad &= M1_{cr} \\
&= 0.0 \text{ k-ft}
\end{aligned}
$$

$$
\begin{aligned}
\text{Right moment} &= M2_{cr} \\
&= C_{cr}(M2_{PScr} + M2_{DLcr}) \\
&= 0.467[6,373 + (-3,899)] \\
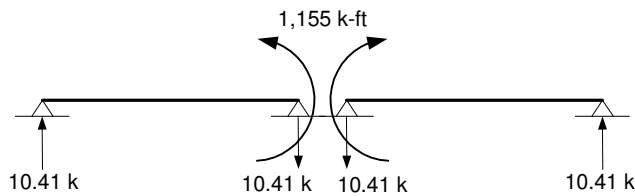&= 1,155 \text{ k-ft}
\end{aligned}
$$

Figure C3 – Total Creep Fixed End Actions

Design Step C1.7

Creep final effects

The fixed end moments shown in Figure C3 are applied to the continuous beam. The beam is analyzed to determine the final creep effects. Due to the symmetry of the two spans of the bridge, the final moments at the middle support are the same as the applied fixed end moments. For a bridge with more than two spans or a bridge with two unequal spans, the magnitude of the final moments would be different from the fixed end moments.
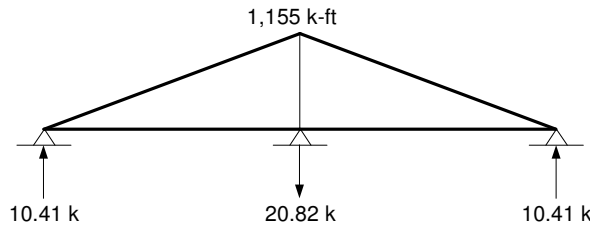

Figure C4 – Creep Final Effects for a Deck and Continuity Connection Cast 450 Days After the Beams were Cast

Design Step C2.1

Analysis of shrinkage effects on the example bridge

Calculate shrinkage strain in beam at infinite time according to S5.4.2.3.3

Calculate the size factor, k_s.

$$k_s = \left[\frac{\left(\dfrac{t}{26e^{0.36(V/S)_b} + t} \right)}{\left(\dfrac{t}{45 + t} \right)} \right] \left[\frac{1{,}064 - 94(V/S)_b}{923} \right] \qquad \text{(SC5.4.2.3.3-1)}$$

where:
  t = drying time
    = infinite days

  e = natural log base (approx. 2.71828)
  $(V/S)_b$ = 4.406 in.

$$k_s = [1]\left[\frac{1{,}064 - 94(4.406)}{923} \right]$$

$$k_s = 0.704$$

Calculate the humidity factor, k_h

Use Table S5.4.2.3.3-1 to determine k_h for 70% humidity, $k_h = 1.0$.

Assume the beam will be steam cured and devoid of shrinkage-prone aggregates, therefore, the shrinkage strain in the beam at infinite time is calculated as:

$$\varepsilon_{sh,b,\infty} = -k_s k_h [t/(55.0 + t)](0.56 \times 10^{-3}) \qquad (S5.4.2.3.3-2)$$

where:
$$k_s = 0.704$$
$$k_h = 1.0 \text{ for 70\% humidity (Table S5.4.2.3.3-1)}$$
$$t = \text{infinite days}$$

$$\varepsilon_{sh,b,\infty} = -(0.704)(1.0)[1](0.56 \times 10^{-3})$$
$$= -3.94 \times 10^{-4}$$

Design Step C2.2

Calculate shrinkage strain in the beam at the time the slab is cast (S5.4.2.3.3)

$$t = \text{time the slab is cast}$$
$$= 450 \text{ days (maximum value)}$$

Calculate the size factor, k_s.

$$k_s = \left[\frac{\left(\dfrac{t}{26e^{0.36(V/S)_b} + t}\right)}{\left(\dfrac{t}{45+t}\right)}\right]\left[\frac{1,064 - 94(V/S)_b}{923}\right] \qquad (SC5.4.2.3.3-1)$$

where:
$$t = 450 \text{ days}$$
$$e = \text{natural log base (approx. 2.71828)}$$
$$(V/S)_b = 4.406 \text{ in.}$$

$$k_s = \left[\frac{\left(\dfrac{450}{26e^{0.36(4.406)} + 450}\right)}{\left(\dfrac{450}{45+450}\right)}\right]\left[\frac{1,064 - 94(4.406)}{923}\right]$$

$$k_s = 0.604$$

Assume the beam will be steam cured and devoid of shrinkage-prone aggregates, therefore, the shrinkage strain in the beam at infinite time is calculated as:

$$\varepsilon_{sh,b,450} = -k_s k_h [t/(55.0 + t)](0.56 \times 10^{-3}) \qquad \text{(S5.4.2.3.3-2)}$$

where:
$$k_s \quad = 0.604$$
$$k_h \quad = 1.0 \text{ for 70\% humidity (Table S5.4.2.3.3-1)}$$
$$t \quad = 450 \text{ days}$$

$$\varepsilon_{sh,b,450} = -(0.604)(1.0)[450/(55.0 + 450)](0.56 \times 10^{-3})$$
$$= -3.01 \times 10^{-4}$$

Design Step C2.3

Calculate the shrinkage strain in the slab at infinite time (S5.4.2.3.3)

Calculate the size factor, k_s

$$k_s \quad = \left[\frac{\left(\dfrac{t}{26 e^{0.36(V/S)_s} + t} \right)}{\left(\dfrac{t}{45 + t} \right)} \right] \left[\frac{1{,}064 - 94(V/S)_b}{923} \right] \qquad \text{(SC5.4.2.3.3-1)}$$

where:
$$t \quad = \text{infinite days}$$

$$e \quad = \text{natural log base (2.71828)}$$

Compute the volume to surface area ratio for the slab.

$$(V/S)_s \quad = (b_{slab})(t_{slab})/(2b_{slab} - w_{tf})$$

where:
$$b_{slab} \quad = \text{slab width taken equal to girder spacing (in.)}$$
$$t_{slab} \quad = \text{slab structural thickness (in.)}$$
$$w_{tf} \quad = \text{beam top flange width (in.)}$$

$$(V/S)_s \quad = 116(7.5)/[2(116) - 42]$$
$$= 4.58 \text{ in.}$$

$$k_s \quad = [1] \left[\frac{1{,}064 - 94(4.58)}{923} \right]$$

$$k_s \quad = 0.686$$

The slab will *not* be steam cured, therefore, use

$$\varepsilon_{sh,s,\infty} = -k_s k_h [t/(35.0 + t)](0.51 \times 10^{-3}) \qquad \text{(S5.4.2.3.3-1)}$$

where:
$k_s = 0.686$
$k_h = 1.0$ for 70% humidity (Table S5.4.2.3.3-1)
t = infinite days

$$\varepsilon_{sh,s,\infty} = -(0.686)(1.0)[1.0](0.51 \times 10^{-3})$$
$$= -3.50 \times 10^{-4}$$

Design Step C2.4

Calculate the differential shrinkage strain as the difference between the deck total shrinkage strain and the shrinkage strain of the beam due to shrinkage that takes place after the continuity connection is cast.

$$\Delta\varepsilon_{sh} = \varepsilon_{sh,s,\infty} - (\varepsilon_{sh,b,\infty} - \varepsilon_{sh,b,450})$$
$$= -3.50 \times 10^{-4} - [-3.94 \times 10^{-4} - (-3.01 \times 10^{-4})]$$
$$= -2.57 \times 10^{-4}$$

Design Step C2.5

Calculate the shrinkage driving end moment, M_s

$$M_s = \Delta\varepsilon_{sh} E_{cs} A_{slab} e' \text{ (from PCA publication referenced in Design Step 5.3.2.2)}$$

where:
$\Delta\varepsilon_{sh}$ = differential shrinkage strain
E_{cs} = elastic modulus for the deck slab concrete (ksi)
A_{slab} = cross-sectional area of the deck slab (in^2)
e' = the distance from the centroid of the slab to the centroid of the composite section (in.)
$= d_{beam} + t_{slab}/2 - NA_{beam\ bottom}$
$= 72 + 7.5/2 - 51.54$
$= 24.21$ in.

$$M_s = (-2.57 \times 10^{-4})(3,834)(116)(7.5)(24.21)$$
$$= 20,754/12$$
$$= 1,730 \text{ k-ft (see notation in Table 5.3-9 for sign convention)}$$

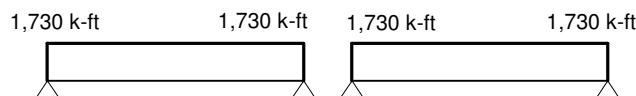1,730 k-ft   1,730 k-ft   1,730 k-ft   1,730 k-ft

Figure C5 – Shrinkage Driving Moment

For beams under constant moment along their full length, the restraint moment may be calculated as shown above for the case of creep due to prestressing force or according to Table 5.3-9.

Shrinkage fixed end actions $= -1.5M_s = -1.5(1,730)$
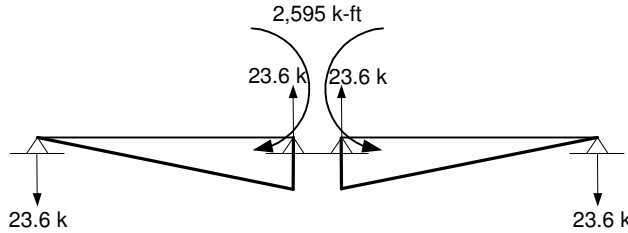$\qquad\qquad\qquad\qquad\qquad\quad = -2,595$ k-ft

Figure C6 – Shrinkage Fixed End Actions

Design Step C2.6   <u>Analyze the beam for the fixed end actions</u>

Due to symmetry of the spans, the moments under the fixed end moments shown in Figure C6 are the same as the final moments (shown in Fig. C7). For bridges with three or more spans and for bridges with two unequal spans, the continuity moments will be different from the fixed end moments.
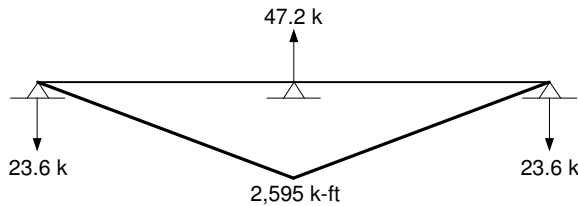

Figure C7 – Shrinkage Continuity Moments

Design Step C2.7   Calculate the correction factor for shrinkage.

$\qquad C_{sh} \quad = (1 - e^{-\phi})/\phi$ (from PCA publication referenced in Step 5.3.2.2)
$\qquad\qquad\quad = [1 - e^{-0.63}]/0.63$
$\qquad\qquad\quad = 0.742$

Design Step C2.8   Calculate the shrinkage final moments by applying the correction factor for shrinkage to the sum of the shrinkage driving moments (Figure C5) and the shrinkage continuity moment (Figure C7) fixed end actions.

End moments, Span 1:

Left end moment $= M1_{sh}$
$= C_{sh}(M_{sh,dr} + \text{shrinkage continuity moment})$
$= 0.742(1{,}730 + 0)$
$= 1{,}284$ k-ft

Right end moment $= M2_{sh}$
$= C_{sh}(M_{sh,dr} + \text{shrinkage continuity moment})$
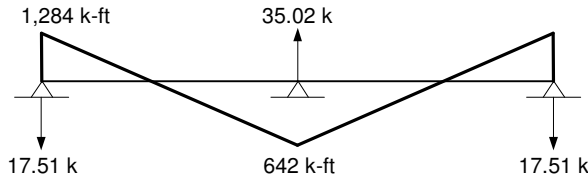$= 0.742(1{,}730 - 2{,}595)$
$= -642$ k-ft

Figure C8 – Final Total Shrinkage Effect

Tables C1 and C2 provide a summary of the final moments for the case of the deck poured 30 days after the beams were cast.

Table C1 - 30 Day Creep Final Moments

Span	M1 (k-ft)	M2 (k-ft)	R1 (k)	R2 (k)
1	0	1,962	-17.7	17.7
2	1,962	0	17.7	-17.7

Table C2 - 30 Day Shrinkage Final Moments

Span	M1 (k-ft)	M2 (k-ft)	R1 (k)	R2 (k)
1	75.9	-37.9	-1.035	1.035
2	-37.9	75.9	1.035	-1.035

When a limit state calls for inclusion of the creep and shrinkage effects and/or the design procedures approved by the bridge owner calls for their inclusion, the final creep and shrinkage effects should be added to other load effect at all sections. The positive moment connection at the bottom of the beams at the intermediate support is designed to account for the creep and shrinkage effects since these effects are the major source of these moments.

Notice that when combining creep and shrinkage effects, both effects have to be calculated using the same age of beam at the time the continuity connection is established.